On nonleptonic decays of Supermultiplets

R. Delbourgo* and Dongsheng Liu**
School of Mathematics and Physics, University of Tasmania,
Hobart, Australia 7001
(March 27, 2009)

Abstract

By describing strong interactions between hadrons via a relativistic supermultiplet scheme and regarding weak interactions as a perturbation thereof, we derive expressions for nonleptonic weak decay amplitudes in terms of constituent quark masses and CKM angles, with no other parameters. Application of this method leads to $\Delta I = 1/2$ dominance in some pseudoscalar meson decays if one scales down the couplings of heavy particles by $\sqrt{M}$ mass factors, in keeping with heavy quark theory expectations. However, certain B and D decay processes to kaons are badly predicted and point to substantial soft gluon renormalization effects in W-quark interactions.

I. INTRODUCTION

With so much data available for the decays of heavy quark composites, a real industry has grown, devoted to calculating nonleptonic weak amplitudes. A standard picture has evolved (Neubert 1994, 1997; Neubert and Stech 1997; Stech 1997) from the work of Bauer, Stech and Wirbel (1987), which parametrises flavour-changing decays in terms of a number of effective four-quark operators, such as

$$\mathcal{H} = \frac{G_F}{2\sqrt{2}} V_{UD} V^*_{ud} [c_1(\mu)(\bar{d}\gamma u)_L.(\bar{U}\gamma D)_L + c_2(\mu)(\bar{U}\gamma u)_L.(\bar{d}\gamma D)_L],$$

and associated parameters $c_i(\mu)$; the idea is then to connect the running $c(\mu)$ values via QCD and the renormalization group with heavy quark theory (Isgur and Wise 1989, 1990) schemes. This description (Ali et al 1998, 1999; Lu 1999) introduces many parameters and requires the input of meson decay constants f (which are extracted from experiment). Even after introducing so many phenomenological constants, a certain amount of confusion remains in this subject, mainly connected with the role of non-factorizable contributions (Beneke 2000, Neubert 2000); indeed some of the elaborations are so Byzantine in their intricacies that one yearns for a simpler and more direct approach to this problem. The only striking and mysterious fact remains the dominance of the $\Delta I = 1/2$ rule in kaon and hyperon decays.

In this paper we will attempt to shed some light on this topic by trying a different approach, one which has long been applied to electromagnetic interactions; namely we will regard the Lagrangian of electroweak theory as a perturbation of the strong interactions. The

difficulty with such an approach lies in knowing the values of strong interaction amplitudes, since this is to be the starting point for applying perturbation theory. Strong amplitudes are of course given reliably by QCD at high energy, but at low energy we are obliged to resort to models which respect the symmetries of QCD, rather than QCD itself, because of unknown effects produced by colour confinement and soft multigluon exchanges that enhance the quark interactions at low mass scales. One of the simplest such models is the linear (or nonlinear) sigma-type model (Scadron 1997) because it embodies full chiral symmetry for massless bare quarks. Another favourite scheme is the spin-flavour supermultiplet model, which not only applies to heavy quarks but to light ones as well (Salam et al 1965; Sakita and Wali 1965), provided their masses are dressed to their constituent values. We explore this route and take it that strong amplitudes are quite well described—often to better than 10%—by supermultiplet tree interactions (Delbourgo and Liu 1996); then we shall add electroweak interactions as a small perturbation[1].

Semileptonic decays are well-understood via weak-boson exchange, provided the hadronic matrix element $\langle f|J^{weak}|i\rangle$ of the weak current J^{weak} is extracted from experiment or estimated reliably from theory, using heavy quark symmetry, dispersion relations, sum rules, $1/N_c$ expansions or whatever other tool one can make use of. We will have nothing to say about semileptonic processes; rather, our focus is on nonleptonic processes where weak and strong interactions are linked. Our attitude towards flavour changing weak amplitudes is that, to first order in the Fermi coupling G_F, the process can be construed as a set of W-exchange loop diagrams between the hadronic participants, with the *hadronic* amplitude satisfactorily determined by higher spin-flavour symmetry interactions. The computational rules are thus fixed and our answers can only depend on the masses of the constituent quarks and the CKM mixing angles. This assumes the Feynman integrals are convergent—which they are, thanks to the unitarity of the CKM matrix[2]. Hence there are no adjustable parameters in our scheme. Note that we do not consider gluon corrections and resultant 'penguin diagrams' in the first place, since gluons are already assumed to be incorporated into dressing the bare quarks and producing the constituent fields which then interact via simple higher symmetry rules.

The layout of the paper is as follows. We summarise the working rules for strong interactions and their weak perturbations in the next section. Then we characterise the types of diagram that should be calculated in the following section. It turns out that there are three types and they are evaluated one by one in Sections IV, V and VI. Next we see to what extent the results are altered by introducing form factors into the weak interactions (Section VII). Finally we apply the ideas to typical pseudoscalar mesons decays, to check whether we are on the right track; we find that the top quark contributions especially are overestimated unless the couplings are scaled down by $1/\sqrt{M}$ factors where M is the mass

[1]A preliminary version of this method was presented in Delbourgo and Liu (1998), but publication requirements meant that the articles were so compressed as to be difficult to follow; the details are fleshed out here and the scope greatly expanded.

[2]But even if they were not, one should be aware that form factors, which are inevitably present, will serve to damp out bare integrals; see section VI.

of the heavy quark, as indicated by heavy quark symmetry. With such damping included, many of the amplitudes fall in the right ballpark—within a factor of two or three. The ability of the scheme to reproduce the '$\Delta I = 1/2$ rule' in most cases is an encouraging sign. However having said that, there are a number of cases in which the supermultiplet predictions are dreadful, indicating that we have overlooked some important feature or that our approach is fundamentally awry: these are processes involving the decays of D-mesons or B-mesons to K-mesons. It is entirely possible that for such heavy/strange objects our hope of absorbing all soft gluon effects into the dressing of the quarks is misplaced, and that they play a very significant role in modifying the weak vertices themselves. If this puzzle can be unravelled, the prospects for calculating other nonleptonic amplitudes, such as initial heavy vector mesons or baryons, are good.

II. STRONG AND ELECTROWEAK INTERACTIONS

The starting point of our model is that hadrons are reasonably well described by multiquark, constituent composites with supermultiplet wavefunctions $A = \alpha a$, where α is a Dirac spinor label and a stands for flavour:

$$\Phi(p)_A^B = [(1+\gamma.v)(\gamma_5 P + \gamma^\mu V_\mu)]_A^B; \quad m = m_a + m_b,\, p = mv \tag{1}$$

$$\begin{aligned}\Psi_{(ABC)}(p) = {} & [(1+\gamma.v)\gamma^\mu C]_{\alpha\beta} u_{\mu(abc)}(p) + \\ & \{[(1+\gamma.v)\gamma_5 C]_{\alpha\beta}\epsilon_{abd} u^d_{c\gamma}(p) + \text{perms}\}; \quad m = m_a + m_b + m_c,\, p = mv\end{aligned} \tag{2}$$

of ground state mesons $(0^-, 1^-)$ and baryons $(1/2^+, 3/2^+)$, with tree-level interactions given by momentum-conserving effective Lagrangians like,

$$g\bar{\Psi}^{(ABC)}\Psi_{(ABD)}\Phi_C^D, \quad f\Phi_A^B\Phi_B^C\Phi_C^A. \tag{3}$$

Such algebraic contractions of indices can be represented pictorially as a joining of the flavour quantum numbers via duality diagrams, with the remaining contractions over spinorial indices serving to provide the Lorentz structure of the amplitude, in keeping with higher symmetry requirements. In (3), with the normalizations used in (1) and (2), the coupling constant g is dimensionless while f has dimensions of mass; we shall return to this point presently.

Now consider the addition of electroweak perturbations on the above sorts of effective Lagrangians, by invoking the standard electroweak Lagrangian,

$$\mathcal{L} = \sum_{\text{flavour}} \bar{\psi}[\gamma.(i\partial + \mathcal{W}) - g_H H]\psi - \vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu}/4 + \mathcal{L}_H + \ldots, \tag{4}$$

where the supermultiplet boson field is

$$\mathcal{W} = -eQA + g_W VW(1-i\gamma_5)/\sqrt{8} + \ldots; \quad V = \text{CKM matrix},\ Q = \text{charge}. \tag{5}$$

By using the Particle Data Group parametrization ($c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij}$),

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\exp(-i\delta) \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\exp(i\delta) & c_{12}c_{23} - s_{12}s_{23}s_{13}\exp(i\delta) & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\exp(i\delta) & -c_{12}s_{23} - s_{12}c_{23}s_{13}\exp(i\delta) & c_{23}c_{13} \end{pmatrix}, \tag{6}$$

one guarantees that unitarity is respected and is not subject to errors by picking specific values for certain matrix elements V_{ij}, without regard to other elements. In practice we shall use the 'average' values, $\theta_{12} = 0.223, \theta_{13} = 0.0031, \theta_{23} = -0.039$ and disregard CP-violation effects which lie outside the scope of this article, by setting $\delta = 0$. In (4) we have not bothered to incorporate the Z-interactions because our main concern is flavour-changing processes. The most significant aspect of (4) is that it is normally applied to current quark fields, not the more massive constituent quark fields which are dynamically induced via strong interactions. It becomes questionable then whether (5) and (6) are applicable to the effective quark fields present in (3). Indeed previous experience shows that we must anticipate nontrivial corrections of around 30% due to renormalization effects; for instance the chiral interaction should be changed from the left-projection $P_L = (1 - i\gamma_5)/2$ to about $(1-\frac{3}{4}\gamma_5)/2$ so that the axial vector component for the nucleon is reduced from 5/3 (arising the higher symmetry D/F ratio of 3/2) to the experimental value of $g_A/g_V \simeq 5/4$. The principal goal of this paper is to see if one can get sensible estimates of *all* nonleptonic amplitudes without invoking extra parameters, so we will ignore relatively small renormalization effects on the axial current but not the more substantial dependence on the mass of the participating constituent quarks in the hadrons. Refinements can come later.

Flavour-changing decay amplitudes are governed by virtual W-exchange between the quarks. We can recognise three types of contribution, where

- a charge-conserving transition occurs on one of the quark lines, either at the start or at the finish (wave-function-like renormalizations) or via a vertex-like correction; see Figures 1a,b,c.

- the charge exchange takes place between the participating quarks in the hadron, again either as a self-energy or a pair of vertex corrections; see Figures 2a,b,c. (If these participating quarks comprise a meson, it must be uncharged.)

- the W acts as a quark annihilation intermediary (see Figure 3) and is just what one encounters in semileptonic processes. It requires a charged meson to be one of the participating particles, of course.

For some processes, there might be a missing type; for example there is no annihilation diagram in $K^0 \to \pi^0\pi^0$ decay.

Since there is a well-defined prescription for treating the strong vertex—where the quark tramlines join up—it only remains to estimate the Feynman integrals corresponding to the W-loop exchange. This we shall presently do, using the Feynman-'tHooft gauge field propagator,

$$\Delta_{\mu\nu}(k) = -\eta_{\mu\nu}/(k^2 - m_W^2).$$

The left matrix $\gamma_{L\mu} = \gamma_\mu(1 - i\gamma_5)/2$ multiplying the propagator has a number of properties which come in handy during the calculations; aside from the obvious utility of the left-handed projection, there is the bonus that Fierz identities can be used to shuffle $\gamma_L \otimes \gamma_L$ from one pair of fermion lines into another pair, thereby relating disparate amplitudes.

This paper will concentrate on the pseudoscalar mesons decays, to verify if the ideas have any sort of validity. We do not seek complete accuracy to any number of decimal places but will content ourselves if we can capture most if not all the amplitudes to within about 30% or better. (If the scheme is successful, the generalisation to vector meson and baryon non-leptonic decays presents itself as the next obvious step.) Throughout we will track flavour indices by making great use of quark line diagrams; sewing the Dirac spinorial indices for each diagram then provides the Lorentz structure. Now, disregarding weak interactions, the three-meson vertex is fixed by the coupling factor f and, because 0^- is forbidden to decay into two other pseudoscalars via strong interactions, one must look to the strong decay $\rho \to 2\pi$, say, in order to get the value of f.

Two quark line diagrams determine $g_{\rho\pi\pi}$ and each of them contains a flavour normalization factor $1/\sqrt{2}$, but with opposite sign. Referring to Figure 4, with all momenta p_i taken as incoming, one therefore arrives at the effective interaction Lagrangian,

$$\begin{aligned}\mathcal{L}_{\rho\pi\pi} &= f\,\mathrm{Tr}[\Phi_P(p_3)\Phi_P(p_2)\Phi_V(p_1)/\sqrt{2} - (1 \leftrightarrow 2)]/4 \\ &= f\,\mathrm{Tr}[(1+\gamma.v_3)\gamma_5(1+\gamma.v_2)\gamma_5(1+\gamma.v_1)\gamma.\epsilon_1 - (1 \leftrightarrow 2)]/4\sqrt{2} \\ &= (p_3-p_2).\epsilon_1 f(m_1+m_2+m_3)/\sqrt{2}m_2m_3 \equiv g_{\rho\pi\pi}(p_3-p_2).\epsilon_1\end{aligned}$$

Experimentally, the ρ-decay width tells us that the dimensionless coupling $g_{\rho\pi\pi} \simeq 6.03$. Thus $f(6\hat{m})/4\hat{m}^2 = \sqrt{2}g_{\rho\pi\pi}$, or $f = \sqrt{8}\hat{m}g_{\rho\pi\pi}/3 \sim 2$ GeV. Now $f(m_1+m_2+m_3)$ is a ubiquitous factor in relativistic supermultiplet theory, so we shall adopt this quantity as the 'universal' value which equals about 4.1 GeV^2, based on the assumption that the quark masses are not far from $m_u \simeq m_d \equiv \hat{m} \sim 0.34$ GeV.

III. TYPE I - QUARK LINE TRANSITIONS

First we shall deal with changes of flavour (but not of charge) on a single quark, which can take place as self-energy-like diagrams at each of the hadronic legs (Figures 1a, 1c) or as a vertex correction across the legs (Figure 1b). In both cases, one must sum over the internal quark flavours; a generic case (Delbourgo and Scadron 1985; Fuchs and Scadron 1986) is the $s-d$ transition, where we have to sum over u, c, t. Quite generally, between i and k quarks, the self-energy part is $\Sigma_{ik}(p) \equiv \sum_j V_i^j V_k^{j*}\Sigma_j(p)$ where

$$\Sigma_j(p) = i\frac{g_W^2}{2}\int \frac{\bar{d}^4k}{k^2-m_W^2}\gamma_{L\mu}\frac{\eta^{\mu\nu}}{\gamma.(p+k)-m_j}\gamma_{L\nu} \equiv p.\gamma_L\mathcal{F}_j(p^2, m_W, m_j). \tag{7}$$

Now $\mathcal{F}$ is potentially troublesome because, *neglecting form factors*, it contains a logarithmic divergence:

$$\mathcal{F}_j = -i\frac{g_W^2}{p^2}\int \frac{\bar{d}^4k\; p.(p+k)}{[k^2-m_W^2][(k+p)^2-m_j^2]} = \int_0^1 dx \int \frac{(1-x)\,\bar{d}^4k}{[k^2+p^2x(1-x)-m_j^2x-m_W^2(1-x)]^2}. \tag{8}$$

However, through unitarity of the CKM matrix and the fact that we are studying flavour changing transitions ($i \neq k$), we can subtract off the dangerous divergent part, contained in

the limit $m_j \to 0$, and happily use the convergent difference instead of $\mathcal{F}$:

$$\mathcal{F}_j(p^2, m_W, m_j) - \mathcal{F}_j(p^2, m_W, 0) = -\frac{g_W^2}{16\pi^2}\int_0^1 dx\,(1-x)\ln\left[1 + \frac{m_j^2 x}{(1-x)(m_W^2 - p^2 x)}\right].$$

This can then be suitably approximated; thus for $p^2 \ll m_W^2$ which applies to all *external* hadrons not containing a top quark,

$$\mathcal{F}_j(m_j) - \mathcal{F}_j(0) \simeq -\frac{g_W^2}{16\pi^2}\int_0^1 dx\; x\ln\left[1 + \frac{m_j^2(1-x)}{x m_W^2}\right] = -\frac{g_W^2}{32\pi^2}\frac{J}{1-J}\left[1 + \frac{J\ln J}{1-J}\right];\; J \equiv \frac{m_j^2}{m_W^2}. \tag{9}$$

In particular, for $m_j \ll m_W$ or $J \ll 1$, we get a very good estimate of (9) with

$$\mathcal{F}_j(m_j) - \mathcal{F}_j(0) \simeq -\frac{g_W^2 J}{32\pi^2} = -\frac{1}{2}\left(\frac{g_W m_j}{4\pi m_W}\right)^2. \tag{10}$$

But for the top quark one must be more careful in approximating (9). Taking the experimental value of m_t and m_W as inputs we estimate

$$\mathcal{F}_t(m_t) - \mathcal{F}_t(0) \simeq -\frac{1.23 g_W^2}{32\pi^2} \simeq -\frac{0.13 g_W^2 m_t^2}{16\pi^2 m_W^2},$$

which is almost exactly 1/4 of the expression (10). Remembering that $G_F/\sqrt{2} = g_W^2/8m_W^2$ $= 8.25\times10^{-6}$ GeV^{-2}, we may finally write the transition element,

$$\Sigma_{ik}(p) \equiv p.\gamma_L \mathcal{F}_{ik} = -\frac{G_F p.\gamma_L}{4\sqrt{2}\pi^2}\sum_j V_i^j V_k^{j*} m_j^2 \rho_j, \tag{11}$$

where the weight factor ρ_q=1 for all but the top quark, when $\rho_t \simeq 1/4$.

Evaluation of the transition factors, $\mathcal{F}_{ik} = -\sum_j V_i^j V_k^{j*}\, G_F m_j^2 \rho_j / 4\sqrt{2}\pi^2$, is reasonably straightforward, using fairly well-known values of mixing angles and the (GeV) values $m_u \simeq m_d \simeq 0.34$,$m_s \simeq 0.48$, $m_c \simeq 1.5$, $m_b \simeq 4.7$, $m_t \simeq 175$. The values are listed in the left-hand columns of Table I, and it should be noted that transition elements for down-type quarks depend significantly on the contribution from the intermediate top quark; the effect is smallest for the $s-d$ transition, but even so the t quark competes well with the u, c contributions; mostly it dominates the other contributions, in spite of the fact that off-diagonal V_t^q terms are quite small.

Next we turn to Figure 1b, which produces the (matrix) vertex integral,

$$\begin{aligned}\Gamma_{ik} &= -i\sum_j V_i^j V_k^{j*}\frac{g_W^2}{2}\int \frac{\bar{d}^4 k}{k^2 - m_W^2}\gamma_{\mu L}\frac{1}{\gamma.(p_k + k) - m_j}\frac{1}{\gamma.(p_i + k) - m_j}\gamma_L^\mu \\ &= i g_W^2 \sum_j V_i^j V_k^{j*}\int \frac{m_j\,(p_k + p_i + 2k).\gamma_L\,\bar{d}^4 k}{[(p_k + k)^2 - m_j^2][k^2 - m_W^2][(p_i + k)^2 - m_j^2]}.\end{aligned} \tag{12}$$

where p_i is the incoming momentum of the i-quark and p_k is the outgoing momentum of the k-quark. Introducing Feynman parameters and assuming that $p_{i,k} \ll m_W$ —as is true for all top-free external hadrons—we may get the one-dimensional parametric representation,

$$\Gamma_{ik} = \sum_j V_i^j V_k^{j*} m_j (p_i + p_k).\gamma_L \frac{g_W^2}{16\pi^2} \int_0^1 \frac{x(1-x)\ dx}{m_W^2 x + m_j^2(1-x)}$$
$$= \sum_j V_i^j V_k^{j*} m_j (p_i + p_k).\gamma_L \frac{g_W^2}{16\pi^2 m_W^2} \left[\frac{J \ln J}{(1-J)^3} + \frac{1+J}{2(1-J)^2}\right]; \quad J \equiv \frac{m_j^2}{m_W^2}. \tag{13}$$

When the intermediate j-quark is considerably lighter than the W, we can make the reasonable approximation, $J \ll 1$ and find

$$\Gamma_{ik} \simeq \frac{1}{2}\left(\frac{g_W}{4\pi m_W}\right)^2 (p_i + p_k).\gamma_L \sum_j V_i^j V_k^{j*} m_j,$$

but for the top quark, the numerical value of $J \sim 4.7$, means that the contribution is about 1/8 of what the light quark approximation above gives. We shall therefore write the vertex correction in the form

$$\Gamma_{ik} \equiv \frac{1}{2}(p_i + p_k).\gamma_L \mathcal{G}_{ik} = \frac{G_F (p_i + p_k).\gamma_L}{4\sqrt{2}\pi^2} \sum_j V_i^j V_k^{j*} m_j \sigma_j, \tag{14}$$

where the weight factor $\sigma_q = 1$ for all quarks but the top, when $\sigma_t \simeq 1/8$. The magnitudes of the vertex transition elements $\mathcal{G}_{ik}$ are listed in the second columns of Table I; the effect from the top is not so significant as for the self-energy elements, but it still dominates the $b \leftrightarrow s, d$ transitions.

We should point out that the above results were derived on the assumption that the W-coupling to the constituent quarks has no form factors; but this assumption is obviously not correct. A more refined calculation ought to include form factors of the pole type $F(k^2) \sim M^2/(M^2 - k^2)$ or something fancier. We shall return to this point later.

It remains to sum up the terms arising from Figures 1a, 1b and 1c. Since hadronic supermultiplets consist of constituent quarks on their mass shells sharing the total momentum according to their mass (they all have equal velocity) with negligible binding, we shall evaluate the result between free spinors $\bar{u}(p_k)..u(p_i)$. In doing so we must be careful to *halve* the self-energy contributions on external quark lines, since they are eventually associated with $Z^{1/2}$ renormalization factors. Hence, between spinors, the sum equals

$$\mathcal{T}_{ik}^I = -\frac{1}{2(\gamma.p_i - m_k)} \Sigma_{ik}(p_i) + \Gamma_{ik}(p_i, p_k) - \Sigma_{ik}(p_k) \frac{1}{2(\gamma.p_k - m_i)}$$
$$= -\frac{\mathcal{F}_{ik}}{2(\gamma.p_i - m_k)} p_i.\gamma_L + \frac{1}{2}(p_i + p_k).\gamma_L \mathcal{G}_{ik} - p_k.\gamma_L \frac{\mathcal{F}_{ik}}{2(\gamma.p_k - m_i)}$$
$$= -\frac{\mathcal{F}_{ik}}{4}\left[1 + i\gamma_5 \frac{m_k - m_i}{m_k + m_i}\right] + \frac{\mathcal{G}_{ik}}{4}\left[(m_k + m_i) + i\gamma_5(m_i - m_k)\right].$$

The last step is to contract $\mathcal{T}^I$ over the hadronic wavefunctions. For three 0^- mesons, the only relevant part of $\mathcal{T}^I$ is the one containing γ_5 so as to get a non-vanishing trace. Therefore, with the flavour labels of Figure 1, we get the generic amplitude:

$$M_{sdcu}^I = f\, \mathrm{Tr}[\Phi_P(p_2)\mathcal{T}^I \Phi_P(p_1)\Phi_P(p_3)]/m_1 m_2 m_3$$
$$= i\frac{(m_d - m_s) f}{4 m_1 m_2 m_3}\left[\mathcal{G}_{sd} + \frac{\mathcal{F}_{sd}}{m_s + m_d}\right] \mathrm{Tr}[(\gamma.p_2 + m_2)(\gamma.p_1 + m_1)(\gamma.p_3 - m_3)]$$
$$= i\frac{f(m_1 + m_2 + m_3)(m_s - m_d)[m_3^2 - (m_1 - m_2)^2]}{2 m_1 m_2 m_3}\left[\mathcal{G}_{sd} + \frac{\mathcal{F}_{sd}}{m_s + m_d}\right]. \tag{15}$$

Recall that the m_i in (14) are the sums of the constituent quark masses comprising the hadron. When analysing any other flavour changing amplitude of type I, it is a simple matter of substituting the appropriate flavour labels in (14) above. We shall frequently be doing so in section VI.

IV. TYPE II - W-EXCHANGE ACROSS QUARKS

We now turn to the graphs contained in Figure 2. The first of these actually corresponds to a $u\bar{c}$ transition into two mesons, dominated by an intermediate $d\bar{s}$ state. Neglecting the small momentum carried by the W meson relative to its mass, this particular contribution is given by the generic contraction,

$$\begin{aligned}
M^{IIA}_{udbsc} &= -\frac{g_W^2\, f}{32m_W^2} V_u^d V_c^{s*} \mathrm{Tr}[(\Phi_P(p_2)(\gamma.p_d+m_d)\gamma_{L\mu}\Phi_P(p_1)\gamma_L^\mu(\gamma.p_s+m_s)\Phi_P(p_3)] \\
&= \frac{g_W^2 m_d m_s f V_d^u V_s^{c*}}{32m_W^2 m_1 m_2 m_3}\mathrm{Tr}\left[(\gamma.p_2-m_2)\left(1+\frac{\gamma.p_1}{m_s+m_d}\right) i\gamma_L.p_1\left(1-\frac{\gamma.p_1}{m_s+m_d}\right)(\gamma.p_3+m_3)\right] \\
&= \frac{iG_F f(m_1+m_2+m_3)m_d m_s V_u^d V_c^{s*}}{2\sqrt{2}m_1 m_2 m_3}\left[1-\left(\frac{m_u+m_c}{m_s+m_d}\right)^2\right](m_d-m_s)(m_1-m_2-m_3), \quad (16)
\end{aligned}$$

because in the intermediate state, $p_d = p_1 m_d/M_1, p_s = -p_1 m_s/M_1; M_1 = m_s + m_d$.

Competing with this answer are the vertex corrections of Figures 2b,2c. The latter involve Feynman integrals which are more difficult to calculate analytically. An interesting technical aspect of the evaluation is that they contain cancelling logarithmic divergences, irrespective of CKM matrix unitarity. Figure 2b and 2c yield, respectively

$$M^{IIB}_{udbsc} = -i\frac{g_W^2 f}{8}\int\frac{V_u^d V_c^{s*}\,\bar{d}^4k}{k^2-m_W^2}\mathrm{Tr}\left[\Phi_P(p_2)\frac{1}{\gamma.(p_u+k)-m_d}\gamma_{L\mu}\Phi_P(p_1)\frac{1}{\gamma.(p_s-k)-m_c}\gamma_L^\mu\Phi_P(p_3)\right], \quad (17)$$

$$M^{IIC}_{udbsc} = -i\frac{g_W^2 f}{8}\int\frac{V_u^d V_c^{s*}\,\bar{d}^4k}{k^2-m_W^2}\mathrm{Tr}\left[\Phi_P(p_2)\gamma_{L\mu}\frac{1}{\gamma.(p_d-k)-m_u}\Phi_P(p_1)\gamma_L^\mu\frac{1}{\gamma.(p_c+k)-m_s}\Phi_P(p_3)\right], \quad (18)$$

where now $p_s = m_s p_3/m_3, p_u = m_u p_1/m_1, p_d = -m_d p_2/m_2, p_c = -m_c p_1/m_1$. Simplifying the sum,

$$M^{IIBC}_{udbsc} = \frac{g_W^2 f}{8}\int\frac{V_u^d V_c^{s*}\,\bar{d}^4k}{k^2-m_W^2}\,\mathrm{Tr}$$
$$\left[\Phi_P(p_2)\left(\frac{1}{\gamma.(p_u+k)-m_d}\frac{\gamma.(k-p_c)-m_s}{(k-p_s)^2-m_c^2}-\frac{\gamma.(k-p_u)-m_d}{(k-p_d)^2-m_u^2}\frac{1}{\gamma.(k+p_c)-m_s}\right)\Phi_P(p_3)\right].$$

This calculation is messy—suggesting numerical methods as a last resort—if all external momenta are religiously kept within the Feynman integral. However, we can achieve an reasonable estimate of the result by going to the soft limit, i.e. neglecting the p-dependence

within the propagators, relative to the large momentum k carried by the W line. Using the supplementary integral,

$$\int \frac{i\,\bar{d}^4k}{k^2-m_W^2}\left[\frac{1}{(k-p_u)^2-m_d^2}-\frac{1}{(k-p_c)^2-m_s^2}\right]=\int_0^1\frac{dx}{16\pi^2}\ln\left[\frac{m_d^2x+m_W^2(1-x)-m_u^2x(1-x)}{m_s^2x+m_W^2(1-x)-m_c^2x(1-x)}\right]$$
$$\simeq\frac{1}{32\pi^2m_W^2}\left[m_c^2+2m_s^2\ln\left(\frac{m_s^2}{m_W^2}\right)-m_u^2-2m_d^2\ln\left(\frac{m_d^2}{m_W^2}\right)\right],$$

valid when $m_i^2 \ll m_W^2$, and keeping leading logarithms, one may estimate

$$M_{udbsc}^{IIBC}\simeq\frac{g_W^2f}{8}\int\bar{d}^4kV_u^dV_c^{s*}\frac{k^2-m_dm_s}{k^2-m_W^2}$$
$$\mathrm{Tr}\left[\Phi_P(p_2)\left(\frac{1}{(k^2-m_c^2)(k^2-m_d^2)}-\frac{1}{(k^2-m_u^2)(k^2-m_s^2)}\right)\Phi_P(p_3)\right]$$

$$=-\frac{iG_FV_u^dV_c^{s*}f}{16\sqrt{2}\pi^2m_2m_3}\left[m_1^2-(m_2+m_3)^2\right]$$
$$\left[(m_u^2-m_c^2)\ln\left(\frac{m_c^2m_u^2}{m_W^4}\right)+\frac{1}{2}(m_s^2-m_d^2)\ln\left(\frac{m_s^2m_d^2m_c^2m_u^2}{m_W^8}\right)\right]. \qquad (19)$$

The significant point about this last result is that it is of the same order of magnitude as M^{IIA}; even though there is a suppression factor of $1/4\pi^2$ from the integration in eq (18), it is compensated by a number of logarithms which are typically in the range $\ln(m_W^2/m_s^2)\sim 10$. In fact, from our viewpoint, cancellations between these sorts of terms are responsible for the small size of the $\Delta I = 3/2$ amplitude in K decays.

V. TYPE III - W-ANNIHILATION

Evidently, this process only applies to charged mesons, which may be incoming or outgoing. A typical example is drawn in Figure 3. It is easy to calculate, from what has gone before. Basically, we take advantage of the fact that through Fierz reshuffling, $(\bar{u}_1\gamma_L^\mu u_2).(\bar{u}_3\gamma_{L\mu}u_4) = -(\bar{u}_3\gamma_L^\mu u_2).(\bar{u}_1\gamma_{L\mu}u_4)$, aside from colour factors; and because we have an extra fermion loop in Figure 3 relative to Figure 2a, the extra (-) sign is effectively swallowed up. Thus, on the assumption that constituent quarks interact left-handedly to a first approximation, Figure 3 gives the same answer as Figure 2a so far as the Lorentz contraction is concerned. Transcribing the flavour labels, and incorporating a colour factor of 3, we get the generic amplitude,

$$M_{sdbcu}^{III}=\frac{3iG_Ff(m_1+m_2+m_3)m_dm_cV_u^sV_c^{d*}}{2\sqrt{2}m_1m_2m_3}\left[1-\left(\frac{m_s+m_u}{m_c+m_d}\right)^2\right](m_d-m_c)(m_1-m_2-m_3). \qquad (20)$$

VI. FORM FACTOR CORRECTIONS

The integrals in sections III-V were derived on the assumption that the coupling of the W-boson to the quarks was pointlike. In fact the weak current must be affected by intermediate vector and pseudoscalar bosons that can latch on to the quark fields and this will produce a natural damping. (This phenomenon is very familiar in QED and is responsible for the finiteness of photonic corrections to the proton neutron mass difference, aside from the contribution due to the intrinsic $u-d$ mass difference. It is typically governed by a strong interaction scale of about 1.1 GeV.) We may estimate the effects of mediating mesons by incorporating the form factor $M^2/(M^2-k^2)$ at each W-leg, where M is some kind of geometric mean of the intermediate mesons on each side of the W-line. The consequence is that the integral (8) say is finite, regardless of CKM unitarity, because it gets modified to

$$\begin{aligned}\mathcal{F}_j &= -i\frac{g_W^2}{p^2}\int\frac{\bar{d}^4k\;p.(p+k)}{[k^2-m_W^2][(k+p)^2-m_j^2]}\left(\frac{M^2}{M^2-k^2}\right)^2\\ &= \frac{g_W^2L^2}{32\pi^2}\left[\frac{1}{1-L}\left(\frac{1}{L}+\frac{\log L}{1-L}+\frac{J^2\log J}{(L-1)(J-1)^2}\right)+\right.\\ &\qquad\left.\frac{J}{(L-J)^2}\left(\frac{1}{J-1}+\frac{J}{L(L-1)}+\frac{J\log(J/L)}{(1-L)^2}+\frac{2J\log J}{(L-1)(L-Q)}\right)\right],\end{aligned} \tag{21}$$

where $L\equiv M^2/m_W^2, J\equiv m_J^2/m_W^2$. The main effect is to enhance the contribution from the top quark; this can be quite substantial and having included it, we must rescale the universal constant f down appropriately so that the plain results (no form factors), which should be governed by G_F, do not go badly out of line. Once this is done there is no more room for maneouvre.

The same sort of modification arises in the vertex integral $\mathcal{G}$ but we shall not bother to exhibit the dependence on L and J because the consequences are very minor; the point is that each $\mathcal{G}$ contribution is given by a well-behaved finite integral. The form factor effects here, in contrast to those on $\mathcal{F}_t$, are very tame and amount to corrections of just a few per cent.

VII. TYPICAL RESULTS AND DIFFICULTIES

Let us now describe some of the consequences of the supermultiplet scheme and the ensuing problems. To keep the discussion clean, we shall ignore channels which involve uncharged mesons that can mix, like η and η', so we will focus on decays where the outgoing particles are pions, kaons and heavy mesons like D and D_s. Below we define M to be the magnitude of the decay amplitude (it has dimensions of mass), which is derived from the partial decay width via

$$\Gamma_{m\to m_1m_2}=|M|^2\Delta/16\pi m^3;\qquad \Delta\equiv\sqrt{[m^2-(m_1-m_2)^2][m^2-(m_1+m_2)^2]}.$$

Throughout we have disregarded the $u-d$ mass difference and used Mathematica to compute the integrals numerically, as required.

First let us consider the time-honoured example of kaon decay. Two typical cases are $K^+ \to \pi^+\pi^0$, corresponding to $\Delta I = 3/2$, and $K_s \to \pi^+\pi^-$ which amounts to $\Delta I = 1/2$. It is of course known that $M^{expt}_{K_s\pi^+\pi^-} = 3.91\times 10^{-7}$ is about twenty times larger than $M^{expt}_{K^+\pi^+\pi^-} = 1.83\times 10^{-8}$ and this is the main feature to be 'explained'. When we tackle these cases via the supermultiplet scheme, we find that with the longer-lived K^+, the type I contributions cancel, as they must, but for the short-lived K^0 there is a significant type I contribution (because the $s-d$ transition is $\Delta I = 1/2$); thus

$$M_{K^0_s\to\pi^+\pi^0} = \sqrt{2}[M^I_{sdud} + M^{II}_{sudud} + M^{III}_{uudsd}]$$

$$M_{K^+\to\pi^+\pi^0} = [M^{II}_{usudu} + M^{III}_{usuud}]/\sqrt{2}$$

have a ratio of about 4, which is still a factor of 5 too small; but this is easily remedied by including form factors[3] using a weak cutoff of about $1.25 m_t$. In this way we can obtain values which are rather close to experiment:

$$|M_{K^0_s\to\pi^+\pi^0}| \simeq 3.9\times 10^{-7}\ \text{GeV};\quad |M_{K^+\to\pi^+\pi^0}| \simeq 1.9\times 10^{-8}\ \text{GeV}.$$

If one turns to D-meson decays, there are some good results, but there are also some dreadful ones. For example, we get the nice answer

$$|M_{D^0\to\bar{K}^0\pi^0}| = |M^{II}_{duucs} - M^{II}_{csddu}|/\sqrt{2} \simeq 1.79\times 10^{-6}\ \text{GeV};\quad \text{cf } |M^{\text{expt}}_{D^0\to\bar{K}^0\pi^0}| \simeq 1.85\times 10^{-6}\ \text{GeV},$$

as well as the ridiculous value (in GeV)

$$|M_{D^+\to\pi^0\pi^+}| = |M^I_{cuud} - M^{II}_{dudcd} - M^{III}_{dsdcu}|/\sqrt{2} \simeq 6.08\times 10^{-5};\ \text{cf } |M^{\text{expt}}_{D^+\to\bar{K}^0\pi^+}| \simeq 1.35\times 10^{-6}.$$

In fact most of the results involving heavy mesons are predicted to be too large in the raw supermultiplet scheme; it is not hard to trace the reason for this effect and thereby cure it.

The point is that the supermultiplet interactions which we wrote down previously did not take account of the $1\sqrt{M}$ diminution of matrix elements which are expected from heavy quark theory, in order to give symmetry results at equal velocity, *not equal momentum*. These factors have little effect on the light quark composites but play a substantial role for the heavy mesons. When the appropriate factors $\sqrt{2m_u/(m_i+m_k)}$ for a meson composed of quarks i and k are incorporated, they depress the nonleptonic decay amplitudes of heavy mesons; thus $M_{D^+\to\pi^0\pi^+}$ goes down to the acceptable value 1.14×10^{-6} GeV and likewise for many other matrix elements. Nevertheless there remain some processes whose reduction is excessive, such as

$$|M_{D^+_s\to K^0\bar{K}^0}| = |M^{II}_{duscs} + M^{III}_{cusds}| \simeq 4.3\times 10^{-9}\ \text{GeV};\ \text{cf } |M^{\text{expt}}_{D^+_s\to K^0\bar{K}^0}| = 2.41\times 10^{-6}\ \text{GeV},$$

and others which stubbornly resist reduction, such as

$$|M_{B^+\to K^0\pi^+}| = |M^I_{sbdu} + M^{III}_{sbduu}| \simeq 2.3\times 10^{-4}\ \text{GeV};\ \text{cf } |M^{\text{expt}}_{B^+\to K^0\pi^+}| \simeq 0.5\times 10^{-7}\ \text{GeV}.$$

[3]and renormalizing the strong coupling because of the enhancement factors.

The cure is therefore only partial and this is most disappointing.

In spite of a number of silly predictions, we believe that our approach makes sound philosophical sense even if the way we have applied the idea has not been wholly successful: weak interactions should be regarded as a perturbation of the strong interactions and not the other way round. Maybe others will find errors with our numerical work and/or will be able to tackle the problem better than we have. We have perhaps been over-ambitious in thinking that we could get away without introducing any parameters and it is possible that with the introduction of many more couplings one can get suitable agreement with all the experimental results: the weak vertices do receive substantial renormalization corrections from the strong interactions. To conclude on a more optimistic note, if one could resolve the difficulties, it would be a simple step to generalize our work to the vector mesons and the baryonic supermultiplet.

Acknowledgments This work was supported by the Australian Research Council under grant number A69800907.

References

* Email: Bob.Delbourgo@utas.edu.au

** Email: D.Liu@utas.edu.au

Ali, A., Kramer, G., and Lu, C-D. (1998).*Phys. Rev.* **D58**, 094009.

Ali, A., Kramer, G., and Lu, C-D. (1999).*Phys. Rev.* **D59**, 014005.

Bauer, M., Stech, B., and Wirbel, M. (1987). *Z. Phys.* **C34**, 103.

Beneke, M. (2000) "Conceptual aspects of QCD factorization in hadronic B decays", hep-ph/0009328.

Delbourgo, R., and Liu, D. (1996). *Phys. Rev.* **D53**, 6576.

Delbourgo, R. and Liu, D. (1998). " Nonleptonic decays: amplitude analysis and supermultiplet schemes", pp 249-260 in 'Nonperturbative Methods in Quantum Field Theory', (ed. by A.W. Schreiber, A.G. Williams and A.W. Thomas), (World Scientific, Singapore).

Delbourgo, R., and Scadron, M.D. (1985). *Nuovo Cim. Lett.* **44**, 193.

Fuchs, N.H., and Scadron, M.D. (1986). *Nuovo Cim.* **A93**, 205.

Isgur, N., and Wise, M.B. (1989). *Phys. Lett.* **B232**, 113.

Isgur, N., and Wise, M.B. (1990). *Phys. Lett.* **B237**, 527.

Lu, C-D. (1999). *Nucl. Phys. Proc. Suppl.* **74**, 227.

Neubert, M. (1994). *Phys. Rept.* **245**, 259.

Neubert, M. (1997). "B-decays and heavy quark expansion" in 'Heavy Flavours II', pp 239-293 (World Scientific, Singapore).

Neubert, M. (2000). "Application of QCD Factorization in Hadronic B-Decays", Cornell preprint.

Neubert, M., and Stech, B. (1997). "Nonleptonic weak decays of B-mesons", in 'Heavy Flavours II', pp 294-344 (World Scientific, Singapore).

Sakita, B., and Wali, K.C. (1965). *Phys. Rev.* **139**, B1355.

Salam, A., Delbourgo, R., and Strathdee, J. (1965). *Proc. R. Soc. London*, **A284**, 146.

Scadron, M.D. (1997). *Nuovo Cim.* **110A**, 865.

Stech, B. (1997). "Twenty Beautiful Years of Bottom Physics", (Chicago Univ. Press), hep-ph/9709280.

TABLES

TABLE I. Flavour changing self-energy transition elements estimated on the basis of eq (10), all in GeV units. The combination $\mathcal{H}_{ij} \equiv \mathcal{G}_{ij} + \mathcal{F}_{ij}/(m_i + m_j)$ is needed in eq (15).

$\mathcal{F}_{sd}$	7.91×10^{-7}	$\mathcal{G}_{sd}$	-1.08×10^{-7}	$\mathcal{H}_{sd}$	2.11×10^{-7}
$\mathcal{F}_{sb}$	-5.96×10^{-5}	$\mathcal{G}_{sb}$	3.17×10^{-7}	$\mathcal{H}_{sb}$	-2.80×10^{-6}
$\mathcal{F}_{db}$	1.86×10^{-5}	$\mathcal{G}_{db}$	-1.00×10^{-7}	$\mathcal{H}_{db}$	9.96×10^{-7}
$\mathcal{F}_{uc}$	-4.30×10^{-9}	$\mathcal{G}_{uc}$	1.15×10^{-8}	$\mathcal{H}_{uc}$	2.29×10^{-9}
$\mathcal{F}_{ut}$	-1.44×10^{-8}	$\mathcal{G}_{ut}$	6.08×10^{-9}	$\mathcal{H}_{ut}$	1.50×10^{-9}
$\mathcal{F}_{ct}$	1.78×10^{-7}	$\mathcal{G}_{ct}$	-6.88×10^{-8}	$\mathcal{H}_{ct}$	-1.70×10^{-8}

FIGURES

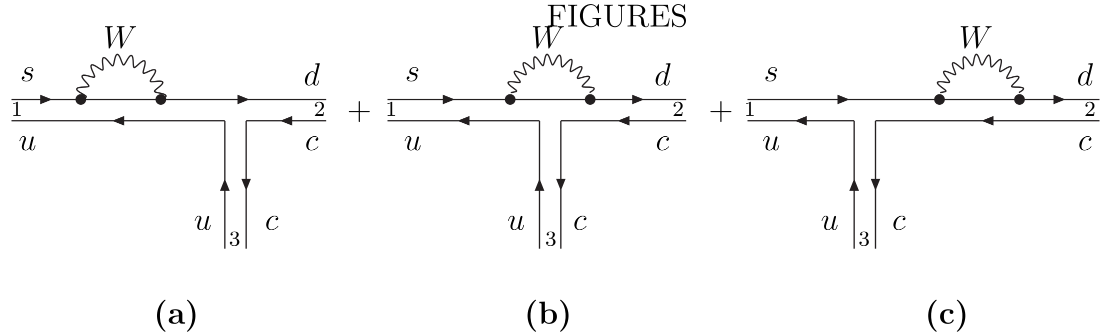

FIG. 1. Single quark line transition.

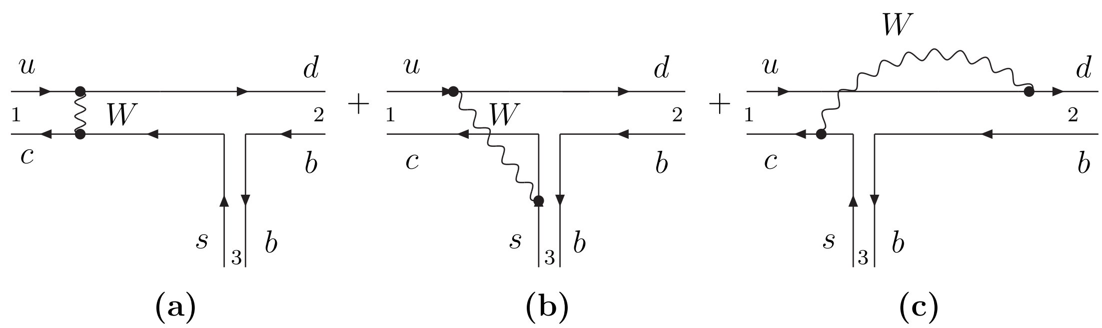

FIG. 2. W-exchange across quark lines.

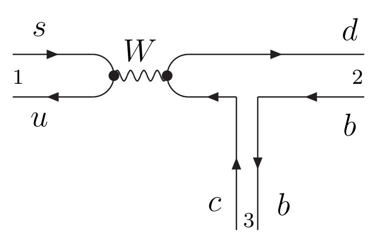

FIG. 3. Annihilation diagram.